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Three dimensional dynamics of ferromagnetic swimmer

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Abstract

It is shown that a flexible ferromagnetic filament self-propels perpendicularly to the AC magnetic field during a limited period of time due to the instability of the planar motion with respect to three dimensional perturbations. The transition from the oscillating U-like shapes to the oscillating S-like shapes is characterized by the calculated Wr number.

Keywords: ferromagnetic filaments, self-propulsion, writhe
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1. Introduction

Ferromagnetic filaments are used by magnetotactic bacteria for the purpose of navigation in the magnetic field of the Earth [1]. They may be created artificially by linking commercially available functionalized ferromagnetic micron size particles with biotinized fragments of DNA [2]. Several phenomena are confirmed experimentally until now - the orientation of the flexible ferromagnetic filament perpendicularly to the AC magnetic field, the formation of loops at sudden field inversion [2]. In [3] by two dimensional numerical simulations it is shown that flexible ferromagnetic filaments should swim perpendicularly to the direction of the AC field due to the periodic formation of the loop and straight line. This begs the question: will such self-propelling motion remain stable with respect to three dimensional perturbations. Here a 3D numerical algorithm taking into account anisotropy of the hydrodynamic drag is developed and it is shown that self-propelling motion persists for a limited period of time after which the filament relaxes to the S-like state oscillating perpendicularly to the AC field and the plane of initial bending.

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2. Model and numerical method

The theoretical model for the dynamics of the ferromagnetic filament is developed in [4, 5] and is based on the expressions for the force $\boldsymbol{F}$ and torque $\boldsymbol{T}$ in the cross section of the filament. In the Frenet frame the relations for $\boldsymbol{F}$ and $\boldsymbol{T}$ reads

$$\boldsymbol{F} = A\frac{d}{dl}\left(\frac{1}{R}\right)\boldsymbol{n} + A\frac{1}{R}\tau\boldsymbol{b} + \left(\Lambda - \frac{1}{2}A\frac{1}{R^2}\right)\boldsymbol{t} + M(\boldsymbol{H} - \boldsymbol{t}(\boldsymbol{t}\cdot\boldsymbol{H}))\ ; \quad (1)$$

$$\boldsymbol{T} = -A\frac{1}{R}\boldsymbol{b}\ . \quad (2)$$

Here $\boldsymbol{t}$ is the tangent vector of center line, $\boldsymbol{n}, \boldsymbol{b}$ are normal and binormal vectors respectively, l is the arc length of the center line, $1/R$ is the curvature of center line and τ its torsion, A is the bending modulus, Λ characterizes the tension arising due to the inextensibility of the filament, $-M\boldsymbol{t}$ is the magnetization per unit length and $\boldsymbol{H}$ is the magnetic field strength.

Using the Frenet equations $d\boldsymbol{t}/dl = -\boldsymbol{n}/R$ and $d\boldsymbol{n}/dl = \boldsymbol{t}/R + \tau\boldsymbol{b}$ the relation (1) may be rewritten as follows

$$\boldsymbol{F} = -A\frac{d^3\boldsymbol{r}}{dl^3} + M\boldsymbol{H} + \tilde{\Lambda}\boldsymbol{t}\ , \quad (3)$$

where $\tilde{\Lambda} = \Lambda - M\boldsymbol{t}\cdot\boldsymbol{H} - 3A/2R^2$. Tildes further are omitted.

The equation of motion taking into account the anisotropy of the hydrodynamic drag reads ($B^{-1} = I + (\zeta_{\parallel} - \zeta_{\perp})\zeta_{\perp}^{-1}\boldsymbol{t}\otimes\boldsymbol{t}$)

$$\zeta_{\perp}B^{-1}\boldsymbol{v} = \frac{d\boldsymbol{F}}{dl}. \quad (4)$$

The filament is represented by $n+1$ marker points at distance h from each other. The condition of inextensibility is imposed by n constraints $g_k = (\boldsymbol{r}_{k+1} - \boldsymbol{r}_k)^2 = h^2 \quad (k = 0, ..., n-1)$. Introducing the $n \times 3(n+1)$ Jacobian matrix J with elements $J = \partial g_k/\partial\boldsymbol{r}_j$ allowed motions satisfy $J\cdot\boldsymbol{v} = 0$. The condition of inextensibility in the discrete form reads

$$\boldsymbol{f}'_0\boldsymbol{v}_0 + \boldsymbol{f}'_n\boldsymbol{v}_n + \sum_{i=1}^{n-1}\boldsymbol{f}'_i\boldsymbol{v}_i = 0, \quad (5)$$

where $\boldsymbol{f}'_0 = \Lambda\boldsymbol{t}\ |_0; \boldsymbol{f}'_n = -\Lambda\boldsymbol{t}\ |_n; \boldsymbol{f}'_i = hd(\Lambda\boldsymbol{t})/dl\ |_i\ (i, 1, ..., n-1)$, which is satisfied for allowed motions if $\boldsymbol{f}' = J^T\cdot\gamma$ (upper script ^{T} denotes transposed matrix, γ is n-dimensional colon vector). Introducing the mobility of marker points $\alpha = (\zeta_{\perp}h)^{-1}$, the notations

$$\boldsymbol{f}^m_0 = M\boldsymbol{H}; \boldsymbol{f}^m_n = -M\boldsymbol{H}; \boldsymbol{f}^m_i = 0\ (i = 1, ..., n-1)$$

and

$$\boldsymbol{f}^e_i = -hAd^4\boldsymbol{r}/dl^4\ |_i; \quad (6)$$
$$\boldsymbol{f}^e_{0,n} = \mp Ad^3\boldsymbol{r}/dl^3\ |_{0,n} \pm M\boldsymbol{H}$$

the equation of motion in the matrix form reads

$$\boldsymbol{v} = \alpha B\cdot(\boldsymbol{f}^e + \boldsymbol{f}^m + \boldsymbol{f}'). \quad (7)$$

Using the condition $J\cdot\boldsymbol{v} = 0$ to express $\boldsymbol{f}'$ and denoting the projection operator on the space of allowed motions as $P = I - B\cdot J^T\cdot(J\cdot B\cdot J^T)^{-1}\cdot J$ the equation of motion is rewritten as follows

$$\boldsymbol{v} = \alpha P\cdot B\cdot(\boldsymbol{f}^e + \boldsymbol{f}^m)\ . \quad (8)$$

Approximating the derivatives by finite differences and introducing the stiffness matrix $\boldsymbol{f} = C\cdot\boldsymbol{r}$ the implicit scheme for the time step τ reads

$$(I - \alpha\tau P_t\cdot B_t\cdot C)\cdot(\boldsymbol{r}_{t+\tau} - \boldsymbol{r}_t) = \alpha\tau P_t\cdot B_t\cdot(C\cdot\boldsymbol{r}_t + \boldsymbol{f}^m). \quad (9)$$

Configuration of the filament at next time step is found by inverting the matrix $I - \alpha\tau P_t\cdot B_t\cdot C$. At the end of each time step the filament is reshaped to satisfy the constraints according to the algorithm described in [6].

The equations are put in dimensionless form by introducing the following scales: time $\zeta_{\perp}(2L)^4/A$; length $2L$, and the elastic force $A/(2L)^3$. The dynamics is controlled by the two parameters: the magnetoelastic number $Cm = MHL^2/A$ and the ratio L/L_e, where $L_e = (a/\omega\zeta_{\perp})^{1/4}$ is the elastic penetration length.

3. Results

The set of the coordinates is chosen as follows. The magnetic field is $\boldsymbol{H} = (H\cos(\omega t), 0, 0)$. Initial configuration initiating the loop formation in x, y plane is given by the equation $y = \varepsilon\sin(\pi x)$, where $x \in [0, x_*]$ and $x_* = 1 - (\varepsilon\pi/2)^2$. For this initial perturbation the filament after some transition period self-propels perpendicularly to the AC field as predicted in [3]. Inducing some perturbation out of the plane of the initial curvature leads to the transition to the S-like state when the filament orients perpendicularly to the field with its ends oscillating in the direction of the AC field [2]. After the transition to this state the symmetry of the filament is restored and its self-propelling motion stops. The time interval during which the quasi-stationary state of the self-propelling motion exists increases with the frequency of the AC field. Dependence of the velocity of the mean self-propelling velocity for small inclination of the initial bent configuration ($\vartheta = z/x = 10^{-4}$) is summarized for $Cm = 72$ in Fig. 1 and shows the maximum at $L/L_e \simeq 5.15$. We should remark that the self-propelling motion perpendicularly to the applied field is not monotonous and the stages of the power stroke and return stroke are present during the one period of the field. This is illustrated by Fig. 2, where displacement of the mass center coordinate perpendicularly to the AC field direction is shown for four periods. Quite remarkable is the fact that period doubling in the sequence of the power and return strokes may be noticed.

Increase of the magnetoelastic number Cm as it is illustrated by Fig. 3 increases the range of the frequency in which self-propelling displacement of the filament in transient stage exists. From the data in Fig. 3 we also see that the frequency at which the mean velocity of the filament is maximal diminishes with the magnetoelastic number. We should remark that the three dimensional instability of self-propelling motion of the ferromagnetic filament does not mean that creation of micro devices using their self-propelling properties is impossible, it only shows that the special care should be taken

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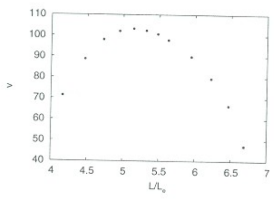

Figure 1: Mean velocity in dependence on the ratio of length to the penetration length. $Cm = 72$, $\vartheta = 10^{-4}$.

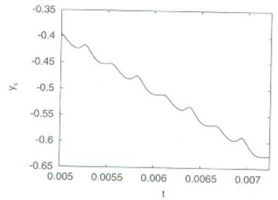

Figure 2: Mass center coordinate y_c in dependence on time for four periods of the AC field. $Cm = 72$, $L/L_e = 5.15$ and $\vartheta = 10^{-4}$.

to the protocol of the field application - for example periodically stabilizing its plane of bending by applying constant field.

The loop formation during the self-propelling motion of the filament is characterized by the Wr number, which is calculated according to the Fuller formula [7]

$$Wr = \frac{1}{2\pi}\int \frac{\mathbf{e}_z(\mathbf{t}\times d\mathbf{t}/dl)}{1+\cos\vartheta}dl\ , \qquad (10)$$

where ϑ is the angle which the tangent to the filament makes with the z axis. As it is shown in Fig. 4 for $L/L_e = 5.15$ the writhe number Wr oscillates during the stage of the self-propelling motion and disappears after the transition to the S-like configuration.

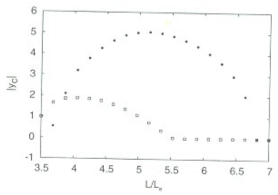

Figure 3: Total displacement of the mass center coordinate y_c in dependence on L/L_e for two values of the magnetoelastic number. $Cm = 72$ (filled circles) and $Cm = 35$ (squares). $\vartheta = 10^{-4}$.

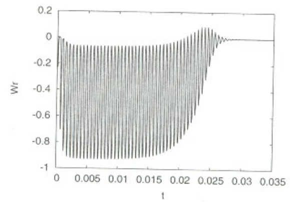

Figure 4: Wr in dependence on time at $Cm = 72$, $L/L_e = 5.15$ and $\vartheta = 10^{-4}$.

4. Conclusions

It is shown that the self-propelling motion of the ferromagnetic filament perpendicularly to the AC magnetic field is destabilized by the three dimensional perturbations which lead to the formation of the symmetric configuration with the average orientation perpendicular to the AC field and the plane of initial bending.

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