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ON MANY–VALUED BORNOLOGICAL STRUCTURES

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In [1] S.T. Hu studied the problem of the possibility to define the concept of boundedness in a topological space. To do this he introduced a system of axioms which later gave rise to the concept of a bornology and a bornological space. In a certain sense a bornological space can be viewed as a counterpart of a topological space if one is mainly interested in the property of boundedness of mappings and not in their property of continuity. At the first stage of research bornological structures were mainly considered on Banach or, more general, on linear topological spaces, but later the reserach was extended to topological spaces without any linear structure, see e.g. [2].

In paper [6] the concept of a bornology on the L-exponent L^X of a set X was introduced and some basic facts about the category of L-bornological spaces and their bounded mappings were proved. The aim of this talk is to introduce the alternative approach to the study of bornological structures in the context of L-sets: Namely we define a many-valued, or, more precisely, an L-valued bornology on the ordinary exponent 2^X of a set X. While the approach in [6] can be considered as a bornological counterpart of Chang-Goguen fuzzy topologies [3], [4], our approach here is a bornological counterpart of Mingsheng Ying's fuzzifying topologies [5].

Definition Let L be a *cl*-monoid, in particular a complete infinitely distributive lattice with lower and upper bounds 0 and 1 respectively . An L-valued bornology on a set X is a mapping $\mathcal{B} : 2^X \to L$, such that (1)$\mathcal{B}(\{x\}) = 1\ \forall x \in X$; (2) $\mathcal{B}(U_1 \wedge U_2) \geq \mathcal{B}(U_1) \wedge \mathcal{B}(U_2)$; (3) $\mathcal{B}(\bigvee_{i\in I} U_i) \geq \bigwedge_{i\in I} \mathcal{B}(U_i) \forall U_i \in 2^X$. Given two L-valued bornological spaces $(X, \mathcal{B}_X, (Y, \mathcal{B}_Y)$, a mapping $f : X \to Y$ is called bounded if $\mathcal{B}_X(U) \leq \mathcal{B}_Y(f(U))\ \forall U \in 2^X$.

We are studying properties of the category $L-\mathbf{BORN}$ of L-valued bornological spaces and their bounded mappings. In particular we show that the family $\mathfrak{B}(X)$ of all L-valued bornologies on a set X ordered in athe natural way is a complete infinitely distributive lattice; prove that the category $L-\mathbf{BORN}$ is topological over the category **SET**; construct intitial and final structures for a family of mappings in this category.

Finally a construction of fuzzification of the category $L-\mathbf{BORN}$ will be briefly mentioned.

REFERENCES

[1] S.-T. Hu . Boundedness in a topological space. *J. Math. Pures Appl.*, **28** 287–320, 1949.

[2] G. Beer. Metric bornologies and Kuratowska-Painleve convergence to the empty set. *Jornal of Convex Analysis.*, **8** 237–289, 2001.

[3] C.L. Chang. Fuzzy topological spaces. *J. Math. Anal. Appl.*, **24** 282–290, 1968.

[4] J.A. Goguen. The fuzzy Tychonoff theorem. *J. Math. Anal. Appl.*, **43** 145–174, 1967.

[5] Ying Mingsheng . A new approach to fuzz topology, Part I. *Fuzzy Sets and systems*, **39** 303–321, 1991.

[6] M. Abel, A.Šostaks. Towards the theory of L-bornological spaces. *Iranian Journal of Fuzzy Systems*, to appear, 2010.