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On bornological-type structures in the context of L-fuzzy sets
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Abstract. In [8], [9] S.T. Hu studied the problem of the possibility to define the concept of boundedness in a topological space. To do this he introduced a system of axioms which later gave rise to the concept of a bornology and a bornological space. In a certain sense a bornological space can be viewed as a counterpart of a topological space if one is mainly interested in the property of boundedness of mappings and not in their property of continuity. At the first stage of research bornological structures were mainly considered on Banach or, more general, on linear topological spaces, but later the reserach was extended to topological spaces without any linear structure, see e.g. [6], [2].

In the paper [1] the concept of a L-bornology (where L is a complete infinitely distributive lattice, or more generally a cl-monoid) on a set X was introduced. An L-bornology on a set X is a subset of the family L^X of its L-subsets $\mathcal{B} \subseteq \mathcal{L}^{\mathcal{X}}$ such that (1) $\mathcal{B}(\S) = \infty\ \forall \S \in \mathcal{X}$, (2) $U \leq V$ and $V \in \mathcal{B} \Longrightarrow \mathcal{V} \in \mathcal{B}\ \forall \mathcal{U}, \mathcal{V} \in \mathcal{L}^{\mathcal{X}}$, (3) $U, V \in \mathcal{B} \Longrightarrow \mathcal{U} \vee \mathcal{V} \in \mathcal{B}\ \forall \mathcal{U}, \mathcal{V} \in \mathcal{L}^{\mathcal{X}}$. A mapping $f : (X, \mathcal{B}_{\mathcal{X}}) \to (\mathcal{Y}, \mathcal{B}_{\mathcal{Y}})$ is called bounded if $f(U) \in \mathcal{B}_{\mathcal{Y}}\ \forall \mathcal{V} \in \mathcal{B}_{\mathcal{X}}$. L-bornological spaces and their bounded mappings form a category L-BORN whose basic properties were first considered in [1].

Further, in our talk at the conference "Mathematical Modelling and Analysis", the concept of a many-valued bornology on a set X was introduced [10]. Actually, a many-valued bornology on a set X is a mapping $\mathfrak{B} : 2^X \to L$ satisfying the following properties (1) $\mathfrak{B}(x) = 1\ \forall x \in X$, (2) $U \subseteq V \Longrightarrow \mathfrak{B}(U) \geq \mathfrak{B}(V)\ \forall U, V \in 2^X$, (3) $\mathfrak{B}(U \cup V) \geq \mathfrak{B}(U) * \mathfrak{B}(V)\ \forall U, V \in 2^X$, where $*$ is an arbitrary t-norm on L, in particular, $* = \wedge$. A mapping $f : (X, \mathfrak{B}_X) \to (Y, \mathfrak{B}_Y)$ is called bounded if $\mathfrak{B}(U) \leq \mathfrak{B}_Y(f(U))\ \forall U \subseteq X$. L-valued bornologies and bounded mappings between them form a category $BORN(L)$.

While category L-BORN is a certain bornological counterpart of the category of L-topological spaces in the sense of Chang-Goguen [3], [4], [5] when introducing the category BORN(L) we had in mind the category of fuzzifying topological spaces in the sense of Hóhle-Ying [7], [11] as its topological analogue.

The aim of this talk to go further in the study of L-bornological and L-valued bornological spaces and the corresponding categories. In particular we show that the families of L-bornologies and L-valued bornologies on a set X ordered in a natural way is a complete infinitely distributive lattice and prove that the categories L-BORN and BORN(L) are topological over the category of sets **SET**.

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