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ON THE CATEGORY OF L-VALUED BORNOLOGICAL SPACES[1]

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In our talk at the previous conference "Mathematical Modelling and Analysis 2010" (Druskininkai, May 2010) we have introduced the concept of an L-valued bornology where $(L, \leq, \bigwedge, \bigvee, *)$ is a cl-monoid, in particular (in case $* = \wedge$) L is a complete infinitely distributive lattice. Namely, by an L-valued bornology on a set X we mean a mapping $\mathcal{B} : 2^X \to L$ such that

1) $\forall x \in X \quad \mathcal{B}(\{x\}) = 1$;

2) If $U \subset V \subset X$ then $\mathcal{B}(V) \leq \mathcal{B}(U)$;

3) $\forall\, U, V \subset X \quad \mathcal{B}(U \cup V) \geq \mathcal{B}(U) * \mathcal{B}(V)$.

A mapping $f : (X, \mathcal{B}_X) \to (Y, \mathcal{B}_Y)$ where $(X, \mathcal{B}_X)$ and $(Y, \mathcal{B}_Y)$ are L-valued bornological spaces is called bounded if $\mathcal{B}_X(A) \leq \mathcal{B}(f(A))$ for every $A \in 2^X$. Some properties of the category of L-valued bornological spaces and their bounded mappings were also discussed in our talk at the International Conference on Topology and Applications in Nafpaktos, Greece in 2010, see [1]

In the present talk we continue the study of L-bornological spaces and their bounded mappings under the additional assumption that L is equipped with an order reversing involution $^c : L \to L$. In particular, we present a construction of an L-valued bornology $\mathcal{B} : 2^X \to L$ from a non-decreasing (by inclusion) family of usual, that is crisp, bornologies $\mathcal{C} = \{C_\alpha, \alpha \in L\}$ on a set X. This construction is given by the equality

$$\mathcal{B}(A) = \bigvee \{\alpha^c \mid A \in C_\alpha\} \quad \forall A \in 2^X.$$

The lattice structure and the categorical properties of this construction will be discussed.

Remark 1. Note that in case $L = \{0, 1\}$ our concept of an L-valued bornology turns into the well known concept of a bornology introduced by S.-T. Hu [2] and thoroughly studied recently by different authors. Note also that the *crisp* bornological-type structure on the family L^X of L-subsets of a set X was introduced and studied in [3].

REFERENCES

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[3] M. Abel and A. Šostak. Towards the theory of L-bornological spaces. *Iranian Journal of Fuzzy Systems*, **8** (1): 19-28, 2011.

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