On the concept of bornology in the context of many-valued sets
Abstract

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The concept of a bornology in the context of fuzzy sets was first introduced in [3]. The aim of this talk is, patterned after [3] to introduce the concept of bornology for the context of many-valued sets and to develop the foundations of the theory of many-valued bornological spaces.

Let $L = (L, \leq, *)$ be a *cl*-monoid [1], whose top and bottom elements are 0 and 1 respectively. Further, let (X, E) be a many-valued set, see e.g. [2] and, let $L^{(X,E)}$ be the family of its extentional L-subsets, that is such L-sets $A : X \to L$ that $A(x) * E(x, x') \leq A(x')\ \forall x, x' \in X$. Given an L-set A let $\tilde{A}$ be its extentional hull, that is the smallest extensional L-set larger or equal than A.

The central concept concerned in this talk is defined below.

An *L-bornology* on a many-valued set (X, E) is a family $\mathcal{B} \subseteq L^{(X,E)}$ such that

(1) $\bigvee\{B \mid B \in \mathcal{B}\} = 1_X$;

(2) $B \in \mathcal{B},\ C \in L^{(X,E)},\ C \leq B \Longrightarrow C \in \mathcal{B}$;

(3) $B_1, B_2 \in \mathcal{B} \Longrightarrow B_1 \vee B_2 \in \mathcal{B}$.

L-bornology $\mathcal{B} \subseteq L^X$ will be called *a strict L-bornology* if it satisfies the following stronger version of the first axiom: $(1')\ \tilde{1}_{\{x\}} \in \mathcal{B}\ \forall\ x \in X$.

The triple $(X, E, \mathcal{B})$ is called *a (strict) many-valued L-bornological space* and L-sets $B \in \mathcal{B}$ are called *bounded* in this space.

Some properties of the category of L-bornological many valued spaces and in a natural way defined morphisms beween such spaces (interpreted as bounded mappings) will be discussed.

References

[1] Birkhoff: *Lattice Theory.* AMS Providence. **RI** (1995).

[2] U. Höhle: *M-valued sets and sheaves over integral commutative cl-monoids,* In: *Applications of Category Theory to Fuzzy Subsets*, S.E. Rodabaugh, E.P. Klement and U. Höhle eds. Kluwer, Dodrecht, Boston. (1992) 33–72.

[3] M. Abel, A.Šostak: *Towards the theory of L-bornological spaces*, Iranian J. of Fuzzy Syst. (to appear).

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