

ON BORNLOGICAL STRUCTURES ON MANY-VALUED SETS¹

INGRĪDA ULJANE

Faculty of Physics and Mathematics, University of Latvia

Zellu iela 8, Rīga LV-1002, Latvia

Institute of Mathematics and Computer Science

Raiņa bulvāris 29, Rīga LV-1459, Latvia

E-mail: ingrida.uljane@lu.lv

The concept of an L-bornology or a fuzzy bornology was first introduced in [3]. The aim of this talk is, patterned after [3], to introduce the concept of an L-bornology in the context of many-valued sets and to start the study of L-bornological structures on many-valued sets.

Let $L = (L, \leq, *)$ be a *cl*-monoid [1], whose top and bottom elements are 0 and 1 respectively. Further, let (X, E) be a many-valued set, see e.g. [2] and, let $L^{(X, E)}$ be the family of its extensional L -subsets, that is L -sets $A : X \rightarrow L$ such that $A(x) * E(x, x') \leq A(x') \forall x, x' \in X$. Given an L -set A let \tilde{A} be its extensional hull, that is the smallest extensional L -set larger or equal than A .

The central concept concerned in this talk is defined below.

An L -bornology on a many-valued set (X, E) is a family $\mathcal{B} \subseteq L^{(X, E)}$ such that

- (1) $\bigvee \{B \mid B \in \mathcal{B}\} = 1_X$;
- (2) $B \in \mathcal{B}, C \in L^{(X, E)}, C \leq B \implies C \in \mathcal{B}$;
- (3) $B_1, B_2 \in \mathcal{B} \implies B_1 \vee B_2 \in \mathcal{B}$.

An L -bornology $\mathcal{B} \subseteq L^X$ will be called a *strict L-bornology* if it satisfies the following stronger version of the first axiom: (1') $\tilde{1}_{\{x\}} \in \mathcal{B} \forall x \in X$.

The triple (X, E, \mathcal{B}) is called a (strict) *many-valued L-bornological space* and L -sets $B \in \mathcal{B}$ are called *bounded* in this space.

Some categorical properties of L -bornological many-valued spaces will be discussed.

REFERENCES

- [1] G. Birkhoff. *Lattice Theory*. AMS Providence. **RI**, 1995.
- [2] U. Höhle. *M-valued sets and sheaves over integral commutative cl-monoids*, In: *Applications of Category Theory to Fuzzy Subsets*, S.E. Rodabaugh, E.P. Klement and U. Höhle eds. Kluwer, Dodrecht, Boston, 1992, 33–72.
- [3] M. Abel and A. Šostak. Towards the theory of L-bornological spaces. *Iranian J. of Fuzzy Syst.*, to appear.

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