

## SOFT NEIGHBORHOOD SETS<sup>1</sup>

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The concept of a soft set was introduced in 1999 by D. Molodtsov [1], see also [2], as a new approach for modelling uncertainties. Somewhat reformulating the original definition, by a *soft set over a set  $X$*  we call a triple  $(M, E, X)$ , where  $E$  is a set interpreted as *the set of parameters* and the mapping  $M : E \rightarrow 2^X$  is referred to as the *soft structure on the set  $X$* .

We consider soft sets as a category **SOFTS** whose objects are soft sets  $(M, E, X)$  and, given two soft sets  $(M, E, X), (N, F, Y) \in \mathcal{Ob}(\mathbf{SOFTS})$ , we take pairs of mappings  $\varphi : X \rightarrow Y$  and  $\psi : E \rightarrow F$  such that  $\varphi^{\leftarrow} \circ M \geq N \circ \psi$  as morphisms  $(\varphi, \psi) : (M, E, X) \rightarrow (N, F, Y)$  in the category **SOFTS**. (Here  $\varphi^{\rightarrow} : 2^X \rightarrow 2^Y$  is the forward powerset operator induced by  $\varphi$ , that is  $\varphi^{\rightarrow}(A) := \varphi(A)$  for each  $A \in 2^X$  (see e.g. [5]).)

The concept of a soft set draw attention both of specialists working in the field of pure mathematics as well as researchers in the area of applied mathematics. This interest was provoked in particular by the fact, that the concept of a soft set is well coordinated with such modern mathematical concepts as a fuzzy set and more general, a many-valued set. In the recent years a series of works was published where *soft generalizations* of different mathematical concepts were introduced and studied. In particular, the concepts of fuzzy soft sets [3], fuzzy soft groups [4], fuzzy soft topologies, fuzzy soft rings, etc., were considered by different authors. On the other hand the aim of our research is *not "a soft generalization"* but rather *"a soft interpretation"* of some known mathematical categories. The base for this interpretation is a special type of a soft set over the powerset  $2^X$  when the set  $X$  is taken as the sets of parameters. In particular, in this talk we characterize topological spaces as follows:

Given a topological *space*  $(X, T)$  we interpret it as a soft *set*  $(\mathcal{U}, X, 2^X)$  over the powerset  $2^X$  where the soft structure  $\mathcal{U} : X \rightarrow 2^{2^X}$  describes the topology  $T$  by a system of neighborhoods of points  $x \in X$ . Further, we characterize the category of topological spaces **TOP** as a subcategory **NSOFTS** (called the category of neighbourhood soft sets) of the category **SOFTS**

Developing this idea we characterize also some other categories related to topology (crisp and fuzzy) as subcategories of the category **SOFTS** of soft sets.

### REFERENCES

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