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ON LATTICES OF L-ROUGH SETS GENERATED BY L-RELATIONS[1]

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L-relations. Recall that a cl-monoid is a tuple $(L, \leq, \wedge, \vee, *)$ where $(L, \leq, \wedge, \vee)$ is a complete lattice with the smallest and the largest elements 0_L and 1_L respectively, and $* : L \times L$ is a commutative associative monotone operation on L which distributes over arbitrary joins, and $1_L * \alpha = \alpha$ for every $\alpha \in L$. By an L-relation on a set X we mean an arbitrary mapping $\rho : X \times X \to L$. An L-relation ρ is called (1) reflexive if $\rho(x,x) = 1$, (2) symmetric if $\rho(x,y) = \rho(y,x)$ and (3) transitive if $\rho(x,y) * \rho(y,z) \leq \rho(x,z)$ for all $x, y, z \in X$. There is a further binary operation - implication $\mapsto$ on a cl-monoid defined by the equality $\alpha \mapsto \beta = \bigvee\{\gamma \mid \gamma * \alpha \leq \beta\}$.

Remark: In case L=[0,1] and $* = \wedge$, L-relations under the name of a fuzzy relation, were first introduced by L.A. Zadeh [1] and later studied by different authors. Obviously in case when L=$\{0,1\}$ the concept of an L-relation turns into the usual, that is crisp, concept of relation.

L-rough sets generated by L-relations. Given $A \in L^X$ and a reflexive, symmetric and transitive L-relation $\rho : X \times X \to L$ we construct an L-rough set $(l_\rho(A), u_\rho(A))$ where $l_\rho \colon L^X \to L^X$ and $u_\rho \colon L^X \to L^X$ are defined by the equalities $l_\rho(A)(x) = \inf_{x' \in X}(\rho(x,x') \mapsto A(x'))$ and $u_\rho(A)(x) = \sup_{x' \in X}(\rho(x,x') * A(x'))$ respectively. The pair of mappings (l_ρ, u_ρ) will be referred to as an approximate system genarated by L-relation ρ.

Remark: In case $L = \{0,1\}$ an L-rough set $(l_\rho(A), u_\rho(A))$ turns obviously into the rough set as it defined in the fundamental Z. Pawlak's work [2] and later studied by different authors. In case $L = [0,1]$ and $* = \wedge$ the concept of an L-rough set under the name of a fuzzy rough set was introduced in the paper [3]. Finally the concept of an L-rough set as it is defined here was described in the paper [4], Section 8.2, by means of the so called approximate systems

Lattice of L-rough sets. Given two L-relations ρ and σ on X we write $(l_\rho, u_\rho) \preceq (l_\sigma, u_\sigma)$ iff $l_\rho(A) \geq l_\sigma(A)$ and $u_\rho(A) \leq u_\sigma(A)$ for every $A \in L^X$. We show that $\rho \leq \sigma \Longrightarrow (l_\rho, u_\rho) \preceq (l_\sigma, u_\sigma)$. This allows to consider the family of all systems (l_ρ, u_ρ), where ρ is an L-relation on X, as a lattice. We study the properties of this lattice. In particular, we show that

$$l_{\cap_{i \in I}\rho_i}(A)(x) \geq \bigwedge_{i \in I} l_{\rho_i}(A)(x),\ l_{\cup_{i \in I}\rho_i}(A)(x) = \bigwedge_{i \in I} l_{\rho_i}(A)(x) \quad \forall A \in L^X,\ \forall x \in X;$$
$$u_{\cap_{i \in I}\rho_i}(A)(x) \leq \bigwedge_{i \in I} u_{\rho_i}(A)(x),\ u_{\cup_{i \in I}\rho_i}(A)(x) = \bigvee_{i \in I} u_{\rho_i}(A)(x) \quad \forall A \in L^X,\ \forall x \in X.$$

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