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AN ANALYSIS OF SMOOTHING–INTERPOLATING PROBLEMS[1]

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The talk deals with the smoothing – interpolating (smoothing for a part of data and interpolating for the rest) problems in the abstract setting of a Hilbert space. Let X, Y be Hilbert spaces and assume that linear operators $T : X \to Y$, $A_1 : X \to I\!R^n$ and $A_2 : X \to I\!R^m$ are continuous. We consider the conditional minimization problem

$$||Tx|| \longrightarrow \min_{x \in H}, \tag{1}$$

where restrictions given by A_1 (interpolating conditions) and A_2 (smoothing conditions) describe the set $H \subset X$.

We use known results separately for the problems of pure interpolation and for the problems of pure smoothing. For a given vector $\boldsymbol{u} \in I\!R^n$ in the case $H = H_1 = \{x \in X: \quad A_1 x = \boldsymbol{u}\}$ a solution of (1) is a spline from the space $S(T, A_1) = \{s \in X: \ < Ts, Tx >= 0 \ \text{ for all } x \in \ \text{Ker} A_1\}$ (called the interpolating spline). For a given vector $\boldsymbol{v} \in I\!R^m$ and parameters $\delta, \varepsilon_i > 0, \ i = 1, \dots, m$, in the case $H = H_2$ or $H = H_3$, where

$$H_2 = \{ \ x \in X: \ \ |(A_2 x)_i - v_i| \le \varepsilon_i, \ i = 1, \dots, m \ \}, \qquad H_3 = \{ \ x \in X: \ \ \sum_{i=1}^{m} ((A_2 x)_i - v_i)^2 \le \delta \ \},$$

a solution of problem (1) is a spline (called the smoothing spline) from the space $S(T, A_2)$. It should be noted that some results proved for smoothing splines in the case $H = H_2$ are true also when $\varepsilon_i = 0$ for some i, i.e. the corresponding interpolation conditions are fulfilled.

In this talk we consider problem (1) with mixed interpolating and smoothing conditions: $H = H_1 \cap H_2$ or $H = H_1 \cap H_3$. The solutions of such problems belong to the space of splines $S(T, A_1 \times A_2)$. We call these splines mixed interpolating – smoothing splines by analogy with the solution of the following conditional minimization problem:

$$||Tg||^2 + \frac{1}{\omega} ||A_2 x - \boldsymbol{v}||^2 \longrightarrow \min_{x \in H_1} \tag{2}$$

where the initial data $\boldsymbol{u}$ are interpolated and the initial data $\boldsymbol{v}$ are smoothed. Note that problem (2), which to a certain extent is connected with problem (1) considered here, was investigated by different authors (A.Y. Bezhaev, V.A. Vasilenko, C. Conti, S.N. Kersey and others).

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