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BOOK of ABSTRACTS

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We consider the question of existence of invariant semidefinite invariant subspaces for J-dissipative operators defined in a Krein space. Recall that a Krein space is a Hilbert space H with an inner product $(\cdot,\cdot)$ in addition endowed with an indefinite inner product of the form $[x,y]=(Jx,y)$, where $J=P^+-P^-$ ($P^\pm$ are orthoprojections in H, $P^++P^-=I$). A subspace M in H is said to be nonnegative (positive, uniformly positive) if the inequality $[x,x]\geq 0$ ($[x,x]>0$, $[x,x]\geq\delta\|x\|^2$ $(\delta>0)$) holds for all $x\in M$. Nonpositive, negative, uniformly negative subspaces in H are defined in a similar way. The main question under consideration here is the question on existence of semidefinite (i. e. of a definite sign) invariant subspaces for a given J-dissipative operator in a Krein space (by a J-dissipative operator we mean an operator dissipative with respect to the indefinite inner product $[\cdot,\cdot]$). We describe some sufficient conditions for a J-dissipative operator in a Krein space to have maximal semidefinite invariant subspaces. The semigroup properties of the restrictions of an operator to these subspaces are studied. Applications are given to the case when an operator admits matrix representation with respect to the canonical decomposition of the space. The main conditions are given in the terms of the interpolation theory of Banach spaces. Given a J-dissipative operator $L:H\to H$ (H is a Krein space), the main conditions look like $(H_1,H_{-1})_{1/2,2}=H$, where $H_1=D(L)$, H_{-1} is a completion of H with respect to the norm $\|(L-\lambda I)^{-1}u\|$, where $\lambda\in\rho(L)$. We present also sufficient conditions ensuring interpolation equalities of this type and some applications to the study of some singular differential operators of the form

$$Lu=\frac{\operatorname{sgn}x}{\omega(x)}(u_{xx}-q(x)u),\ \ x\in\mathbb{R}.$$

Assuming that

(A) $\omega,q\in L_{1,loc}(\mathbb{R})$, $\omega>0$ and $q\geq 0$ almost everywhere on $\mathbb{R}$ and $\omega\notin L_1(\mathbb{R})$,

(B) there exists a constant $\delta>0$ such that $q+\omega\geq\delta/(1+x^2)$,

(C) $\mu(\{x\in\mathbb{R}:q(x)\neq 0\})>0$,

and some regularity of the weight function ω near zero, we prove that the operator L (which is J-selfadjoint J-nonpositive in a Krein space $L_{2,\omega}(\mathbb{R})$ with the norm $\|u\|=\|\sqrt{\omega}u\|_{L_2(\mathbb{R})}$ and $Ju=\operatorname{sgn}u$) has maximal nonnegative and nonpositive invariant subspaces and is similar to a selfadjoint operator in $L_{2,\omega}(\mathbb{R})$.

Reduction principle in the theory of stability of impulsive equations

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Consider the following system of impulsive differential equations in Banach space $\mathbf{X}\times\mathbf{Y}$:

$$\begin{cases} dx/dt &= A(t)x+f(t,x,y), \\ dy/dt &= B(t)y+g(t,x,y), \\ \Delta x\big|_{t=\tau_i} &= x(\tau_i+0)-x(\tau_i-0) = C_ix(\tau_i-0)+p_i(x(\tau_i-0),y(\tau_i-0)), \\ \Delta y\big|_{t=\tau_i} &= y(\tau_i+0)-y(\tau_i-0) = D_iy(\tau_i-0)+q_i(x(\tau_i-0),y(\tau_i-0)), \end{cases} \tag{1}$$

satisfying the *conditions of separation*

$$\nu=\max\left(\sup_s\left(\int_{-\infty}^s|Y(s,t)||X(t,s)|\,dt+\sum_{\tau_i\leq s}|Y(s,\tau_i)||X(\tau_i-0,s)|\right),\right.$$

$$\sup_s \left(\int_s^{+\infty} |X(s,t)||Y(t,s)|\, dt + \sum_{s<\tau_i} |X(s,\tau_i)||Y(\tau_i - 0, s)| \right) \right) < +\infty,$$

and $f(t,\cdot), g(t,\cdot), p_i, q_i$ are ε-Lipshitz, $f(t,0,0) = p_i(0,0) = 0$, $g(t,0,0) = q_i(0,0) = 0$.

Theorem 1. *If $4\varepsilon\nu < 1$, then there exists a unique piecewise continuous map with respect to t satisfying the following properties: $u(t, x(t,s,x,u(s,x))) = y(t,s,x,u(s,x))$ for $t \geq s$, $|u(s,x) - u(s,x')| \leq k|x - x'|$ and $u(t,0) = 0$.*

Theorem 2. *Let $4\varepsilon\nu < 1$. Then for every solution $(x(\cdot), y(\cdot))\colon [s,+\infty) \to \mathbf{X} \times \mathbf{Y}$ of the impulsive system (1) there is such solution $\zeta(\cdot)\colon [s,+\infty) \to \mathbf{X}$ of the impulsive system*

$$\begin{cases} dx/dt &= A(t)x + f(t,x,u(t,x)), \\ \Delta x\big|_{t=\tau_i} &= C_i x(\tau_i - 0) + p_i(x(\tau_i - 0), u(\tau_i - 0, y(\tau_i - 0))) \end{cases} \tag{2}$$

that for all $t \geq s$ fulfils the estimate $|\zeta(t) - x(t)| \leq k_1 |y(t) - u(t,x(t))|$.

We assume in addition that

$$\mu = \sup_s \left(\int_s^{+\infty} |Y(t,s)|\, dt + \sum_{\tau_i > s} |Y(\tau_i - 0, s)| \right) < +\infty.$$

Theorem 3 (Reduction principle). *Let $4\nu\varepsilon < 1$ and $2\varepsilon\mu < 1 + \sqrt{1 - 4\varepsilon\nu}$. The trivial solution of impulsive system (1) is integrable stable, integrable asymptotically stable or integrable nonstable if and only if the trivial solution of impulsive system (2) is integrable stable, integrable asymptotically stable or integrable nonstable.*

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References

[1] Pliss V. A., *A reduction principle in the theory of stability of motion*, Izv. Akad. Nauk SSSR Ser. Mat. 28 (1964), no. 6, 1297–1324 (Russian).

[2] Reinfelds A., Janglajew K. *Reduction principle in the theory of stability of difference equations*, Discrete Cont. Dyn. Syst. Suppl. (2007), 864-874.

On nonlinear heat conduction in doubly periodic 2D composite materials

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It is given an analytic solution to heat conduction problem in 2D unbounded doubly periodic composite materials with temperature dependent conductivities of its components (matrix and inclusions). Linear boundary value problem for a quasi-linear differential equation is reduced to the non-linear boundary value problem for Laplace equation. By introducing complex potentials, the later is reduced to a nonlinear boundary value problem for doubly periodic analytic functions. This problem is investigated via application of a combination of the method of functional equations and the method of the successive approximation. Detailed description of a new algorithm for the construction of any level approximate solution to the starting problem is given.

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