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METHOD OF LINES AND FINITE DIFFERENCE SCHEMES OF EXACT SPECTRUM FOR SOLUTION THE HYPERBOLIC HEAT CONDUCTION EQUATION

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The solutions of corresponding 1-D initial-boundary value problem and inverse problem for hyperbolic heat conduction equation are obtained numerically, using for approach differential equations the discretization in space applying the finite difference scheme (FDS) and the difference scheme with exact spectrum (DSES) [1]. The solution in the time is obtained analytically and numerically with continuous and discrete Fourier methods.
Using the spectral method are obtained news transcendental equation and algorithms for obtaining the last eigenvalue and eigenvector of finite difference scheme.
We define the DSES, where the finite difference matrix A is represented in the form form $A = PDP^T$ (P, D is the matrixes of finite difference eigenvectors and eigenvalues correspondently) and the elements of diagonal matrix D are replaced with the first eigenvalues from the differential operator.
We consider the following hyperbolic heat conduction problem :

$$\begin{cases} \tau \frac{\partial^2 T(x,t)}{\partial t^2} + \frac{\partial T(x,t)}{\partial t} = \frac{\partial}{\partial x}(\bar{k}\frac{\partial T(x,t)}{\partial x} + f(x,t)), x \in (0,L), t \in (0,t_f), \\ \frac{\partial T(0,t)}{\partial x} - \alpha_1(T(0,t) - T_l) = 0, \frac{\partial T(L,t)}{\partial x} + \alpha_2(T(L,t) - T_r) = 0, t \in (0,t_f), \\ T(x,0) = T_0(x), \frac{\partial T(x,0)}{\partial t} = V_0(x), x \in (0,L), \end{cases} \tag{1}$$

where $\bar{k}$ is the heat conductivity, t_f is the final time, τ is the relaxation time ($\tau < 1$), $T_l, T_r, T_0(x), f(x,t)$ are given functions, α_1, α_2 are the heat transfer coefficients (for boundary conditions of first kind $\alpha_1 = \alpha_2 = \infty$)
For the inverse problem the function $V_0(x)$ is unknown and then we can used the aditional condition $T(x,t_f) = T_f(x)$, where T_f is given final temperature.

For finite difference approximation with central differences strong numerical oscillations are presented, when the initial and boundary conditions are discontinuous [2]. The method of DSES is without oscillations and this is effective for numerical solutions[1].

REFERENCES

[1] V.L. Makarov, I.P. Gavrilyuk. On constructing the difference net circuits with the exact spectrum. *Dopov. Akad. Nauk Ukr. RSR,Ser. A*, 1077–1080, 1975. (in Ukrainian)

[2] R.Ciegis. Numerical solution of hyperbolic heat conduction equation. *Mathematical Modelling and Analysis*, **14** (1):11–24, 2009.

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