

INVOLVING FUZZY ORDER IN THE DEFINITION OF MONOTONICITY FOR THE AGGREGATION PROCESS¹

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Since the introduction of the concept of a fuzzy set by L. A. Zadeh [4] and its generalization by J. A. Goguen [2], fuzzy analogues of basic concepts of classical mathematics were introduced and investigated, fuzzy relations [5] among them. In the last years theoretical results obtained in the theory of fuzzy relations were involved for solving problems of practical nature (see eg.[1]). The aim of this work is to involve fuzzy order relation in the study of aggregation process (see eg.[3]). Namely, we use the fuzzy order relation instead of the crisp order relation in the definition of monotonicity. Recall that aggregation function is a mapping satisfying boundary conditions and the condition of monotonicity. In this work we focus only on the condition of monotonicity and define the degree of monotonicity in the following way:

DEFINITION 1. Let $f : [0, 1]^n \rightarrow [0, 1]$ be a function (aggregation function), $P : [0, 1]^2 \rightarrow [0, 1]$ be a fuzzy order relation and \mapsto_T the residuum corresponding to a t-norm $T : [0, 1]^2 \rightarrow [0, 1]$. We define the degree of monotonicity for a function (aggregation function) f w.r.t fuzzy relation P and residuum \mapsto_T in the following way:

$$M_{P, \mapsto_T}(f) = \inf_{x, y} (\bigwedge_i P(x_i, y_i) \mapsto_T P(f(x), f(y))).$$

After giving main definitions we illustrate the introduced notions by examples and study the properties of aggregation functions which have a certain degree of monotonicity.

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