# Categorical aspects of aggregation of fuzzy relations

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In our work we consider aggregation of fuzzy relations from the categorical point of view. Namely we construct a fuzzy category $\mathcal{C}$, whose objects are sets with fuzzy relations and whose morphisms are functions which preserve properties of fuzzy relations. For example: fuzzy POS category with fuzzy partially ordered sets as objects and "potential" order preserving mappings as morphisms. We enrich this category with an $L$-fuzzy subclass of the class of morphisms which is a mapping from the class of morphisms to a commutative cl-monoid $L$: $\mu : MOR(\mathcal{C}) \to L$. The intuitive meaning of the value $\mu(f)$ where $f : (X, P_X) \to (Y, P_Y)$ is the degree to which a morphism $f$ preserves the properties of the object $(X, P_X)$. In case of the fuzzy POS category $\mu(f)$ characterizes the degree to which $f$ is an order-preserving mapping, or, in other words, it shows how good does the morphism $f$ preserve reflexivity, transitivity and antisimmetry of the relation. (For the concept of a fuzzy category see [3],[4]).
We continue by constructing an aggregation model in this fuzzy category. We use the following definition for aggregation of fuzzy relations:

**Definition 1** *Let $A$ be an aggregation operator and let $R_1, R_2, ..., R_n$ be fuzzy relations ($R_i : X \times X \to [0, 1]$). An aggregation fuzzy relation $R_A : X \times X \to [0, 1]$ is defined by the formula*

$$R_A(x, y) = A(R_1(x, y), ..., R_n(x, y)),\ x, y \in X.$$

There are works where the problem which aggregation operators preserve properties of fuzzy relations in the aggregation process are studied. (see e.g. [1],[2]). On other hand our aim here is to involve the abovementioned concept of the degree $\mu$ in order to estimate to what extent do the aggregation operators preserve properties of fuzzy relations.

## References

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**Acknowledgement** This work was partially supported by ESF research project 2009/0223/1DP/1.1.1.2.0/09/APIA/VIAA/008.

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