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HIGHER ORDER FINITE DIFFERENCE SCHEMES FOR PERIODICAL BOUNDARY CONDITIONS[1]

AIGARS GEDROICS[1] and HARIJS KALIS[1,2]

[1] *Department of Mathematics, University of Latvia*
Zeļļu iela 8, Rīga, LV-1002, Latvia

[2] *Institute of Mathematics and Computer Science of University of Latvia*
Raiņa bulvāris 29, Rīga, LV-1459, Latvia

E-mail: aigors@inbox.lv, kalis@lanet.lv

For approximation of the operator $-\frac{\partial^2}{\partial x^2}$, $(x \in [0, L])$ with periodical boundary conditions finite difference expressions with the different order of approximation are investigated. We consider corresponding discrete spectral problem $Ay = \mu y$ for finite difference operator on uniform grid $x_j = jh, j = \overline{1,N}, Nh = L$ where h is the grid parameter, μ is eigenvalue and A, y are the circulant matrix and column-vector of N order (eigenvector) with elements $y_j, j = \overline{0,N}$.

We use from two vectors y^1, y^2 following scalar product $\left[y^1, \bar{y}^2\right] = h\left(\sum_{j=1}^{N} y_j^1 \bar{y}_j^2\right)$, where $\bar{y}$ is the conjugate value of y. The corresponding discrete spectral problem $Ay^n = \mu_n y^n$, $n = \overline{1,N}$ with circulant matrix A have following eigenvectors: $y^n = C_n^{-1}(y_1^n, y_2^n, \ldots, y_N^n)^T$, where $y_j^n = \exp(2\pi i n x_j/L)$, $j = \overline{1,N}$, $i = \sqrt{-1}$ are the components of column-vector y^n. The constants $C_n = \sqrt{N}$ and we have the orthonormed eigenvectors y^n.

We obtain the matrix A and eigenvalues μ_n of matrix A for different order of approximation $O(h^k), k \geq 2$:

1. $k = 2, h^2A = [2, -1, 0, \cdots, 0, -1]$, $h^2\mu_n = 4\sin^2(\pi n/N)$,
2. $k = 4, h^2A = [\frac{5}{2}, -\frac{4}{3}, \frac{1}{12}, 0, \cdots, 0, \frac{1}{12}, -\frac{4}{3}]$, $h^2\mu_n = 4(\sin^2(\pi n/N) + \frac{1}{3}\sin^4(\pi n/N))$,
3. $k = 6, h^2A = [\frac{49}{18}, -\frac{3}{2}, \frac{3}{20}, -\frac{1}{90}, 0, \cdots, 0, -\frac{1}{90}, \frac{3}{20}, -\frac{3}{2}]$, $h^2\mu_n = 4(\sin^2(\pi n/N) + \frac{1}{3}\sin^4(\pi n/N) + \frac{8}{45}\sin^6(\pi n/N))$,
4. $k = 8, h^2A = [\frac{205}{72}, -\frac{8}{5}, \frac{1}{5}, -\frac{8}{315}, \frac{1}{560}, 0, \cdots, 0, \frac{1}{560}, -\frac{8}{315}, \frac{1}{5}, -\frac{8}{5}]$,
   $h^2\mu_n = 4(\sin^2(\pi n/N) + \frac{1}{3}\sin^4(\pi n/N) + \frac{8}{45}\sin^6(\pi n/N) + \frac{4}{35}\sin^8(\pi n/N))$, etc.

Therefore the matrix A can be represented in the form $A = PDP^*$, where the column of the matrix P and the diagonal matrix D contains N orthonormed eigenvectors y^n and eigenvalues $\mu_n, n = \overline{1,N}$ correspondly. From $P^*P = E$ follows that $P^{-1} = P^*$, where E is the unit matrix. For solving the problems of the mathematical physics we compare these methods with the scheme with the exact spectrum, when in the matrix D elements are the first N eigenvalues of the continuous differential operator $\lambda_n = (\frac{2n\pi}{L})^2$.

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