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NUMERICAL SIMULATION FOR SOME HEAT TRANSFER EQUATION WITH PERIODICAL BOUNDARY CONDITION[1]

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The model of finding approximate solution of the partial differential equation

$$U_t = U_{xx}, x \in (0,1) \tag{1}$$

with periodical boundary conditions and initial condition

$$U(0.t) = U(1,t),\ U_x(0,t) = U_x(1,t),\ U(x,0) = U_0(x) \tag{2}$$

is created. At first the equation (1) is approximated in uniform grid with step h dividing the x interval $(0,1)$ into n equal parts. It results equation

$$u' = Au \tag{3}$$

In the case of five-point stencil the matrix A would be $n \times n$ circulant matrix with the first row

$$(c_0, c_1, \cdots, c_{n-1}) := \frac{1}{12h^2}(-30, 16, -1, 0, 0, \cdots, 0, -1, 16) \tag{4}$$

Using the properties of the circulant matrices the approximate solution of the system (1), (2) is found

$$u(x,t) = \sum_{k=0}^{n-1} h(U_0(x), \overline{\phi_k}) e^{f(\omega^k)t} \phi_k \tag{5}$$

ω is the nth primitive root of unity, $f(\lambda) = \sum_{i=0}^{n-1} c_i \lambda^i$, and $\phi_k = (1, \omega^k, \omega^{2k}, \cdots, \omega^{(n-1)k})^T$ – the eigenvectors of the matrix A.

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