

MODULARITY IN THE SEMILATTICE OF ω WORDS¹

JĀNIS BULS

Faculty of Physics and Mathematics, University of Latvia

Zellu 8, Rīga, LV 1002, Latvia

E-mail: buls@mf.lu.lv

Closure properties of some classes of ω -words have been studied extensively (see for example [4; 5; 6]). We are interested in the general algebraic structure of ω -words.

Mealy machines are a simple model of a word transforming automaton with the beneficial property of always transforming an ω -word into an ω -word. A partial ordering is defined on ω -words by looking whether word x can be transformed into word y by transforming it with a Mealy machine (we write $x \rightarrow y$). When both $x \rightarrow y$ and $y \rightarrow x$ are true, we say that x and y are machine invariant.

Buls [2] has shown that machine invariant classes of ω -words form a completely distributive lattice. Belovs [1] showed that the machine-transformation poset of ω -words is a join-semilattice and that the width of this join-semilattice is continuum while the depth is at least \aleph_0 . Buls and Cers [3] announced that this join-semilattice is not distributive. We show that this join-semilattice is not modular too.

REFERENCES

- [1] A. Belovs. Some algebraic properties of machine poset of infinite words. *J. RAIRO-Theor. Inf. Appl.*, **42** 451-466, 2008.
- [2] J. Buls. Machine Invariant Classes. In: *TUCS General Publications*, 27, 207-211, 2003.
- [3] J. Buls, E. Cers. The semilattice of ω -words. <http://aaa79.inf.upol.cz/AAA79Program.pdf>, 2010.
- [4] A. Cobham. Uniform tag sequences. *J. Math. Systems Theory*, **6** 164-192, 1972.
- [5] F. M. Dekking. Iteration of maps by an automaton. *J. Discrete Math.*, **126** 81-86, 1994.
- [6] A. Muchnik, A. Semenov and M. Ushakov. Almost periodic sequences. *J. Theoretical Computer Science*, **304** 1-33, 2003.

¹This work was supported by the ESF project Nr. 2009/0223/1DP/1.1.1.2.0/09/APIA/VIAA/008.