Abstracts of MMA2011, May 25–28, 2011, Sigulda, Latvia

A MATHEMATICAL MODEL FOR A NORWEGIAN FLUTE[1]

TABITA BOBINSKA[1] and ANDRIS BUIKIS[1,2]

[1] *Department of Mathematics, University of Latvia*

Zeļļu iela 8, Rīga, LV-1002, Latvia

[2] *Institute of Mathematics and Computer Science of University of Latvia*

Raiņa bulvāris 29, Rīga, LV-1459, Latvia

E-mail: tabita.bobinska@gmail.com, buikis@latnet.lv

The Norwegian folk flute, also known as willow flute (Norwegian: *seljefløyte*), is a musical instrument of the recorder family, although it is side-blown. It consists of a tube with a transverse fipple mouthpiece that provides a passageway for airflow to be directed against a sharp edge creating a tone. The flute has no finger holes, so different pitches are produced by overblowing and by using a finger to cover, half-cover or uncover the hole at the far end of the tube.

As it is well known, string instruments and many wind instruments are modelled by the wave equation (see, e.g., [1] - [5], etc.). But unlike the authors who use the homogeneous one-dimensional equation to describe sound vibrations inside the willow flute (see [1], [4]), we describe the process more accurately by adding a source term to the equation:

$$\frac{\partial^2 u}{\partial t^2} = a^2 \frac{\partial^2 u}{\partial x^2} + f(x,t),\ 0 < x < l.$$

Here $u(x,t)$ denotes the pressure in the flute, x is the distance along the tube, t is time.

To state the initial-boundary value problem, the following boundary conditions are introduced:

$$u_x(0,t) - \gamma u(0,t) = 0,$$
$$(1-h)u_x(l,t) + hu(l,t) = 0,\ 0 \le h \le 1.$$

Solving the problem, we find the possible frequencies of the harmonics of sounds produced by the flute, determine the distribution of energy between the fundamental and its overtones, and analyse how the tones may be altered by changing the parameter h in the boundary condition at $x = l$.

REFERENCES

[1] R. W. Hall and K. Josić. The Mathematics of Musical Instruments. *American Mathematical Monthly*, **108** (4): 347–357, 2001.

[2] H. Lamb. *The Dynamical Theory of Sound*. Edward Arnold, London, 1910.

[3] T. D. Rossing (Ed.). *Springer Handbook of Acoustics*. Springer Science+Business Media, LLC New York, 2007.

[4] L. Simanoviča. *Boundary Conditions Effect on Musical Instruments Timbre: Bachelor's Thesis*. Faculty of Physics and Mathematics, University of Latvia, Riga, 2010.

[5] J. W. Strutt, Baron Rayleigh. *The Theory of Sound, Vol. II*. Macmillan and Co., London, 1878.

[1] This work is partially supported by the project 2009/0223/1DP/1.1.1.2.0/09/APIA/VIAA/008 and the project "Support for Doctoral Studies at University of Latvia" of the European Social Fund and by the grant 09.1572 of the Latvian Council of Science.