Abstracts of MMA2011, May 25–28, 2011, Sigulda, Latvia

STRICTLY CONVERGENT ALGORITHM FOR AN ELLIPTIC EQUATION WITH NONLOCAL AND NONLINEAR BOUNDARY CONDITIONS[1]

K. BIRGELIS and U. RAITUMS

Department of Mathematics, University of Latvia

Zeļļu iela 8, Rīga, LV-1002, Latvia

E-mail: uldis.raitums@lumii.lv

We consider an elliptic equation with nonlinear and nonlocal boundary conditions, which arises in conductive-radiative heat transfer problems, see, for instance, [1; 2; 3]. The corresponding to our problem variational equality reads as

$$\begin{aligned} &\int_\Omega \big[k_1\langle\nabla(u+u_*),\nabla\eta\rangle + k_2(u+u_*)_{x_3}\eta\big]\,dx \\ &\quad+\int_\Gamma \sigma\big[\big(I-H\big)(\mid u+u_*\mid^3 (u+u_*))\big]\eta\,dS \\ &\quad=\int_\Omega\langle \overline{f},\eta\rangle\,dx + \int_\Gamma g\eta\,dS \quad \forall\eta\in V, \end{aligned} \tag{1}$$

where $\Omega=\Sigma\times[0,L]\subset\mathbf{R}^3$ is a bounded cylinder , V is a subspace of $W_2^1(\Omega)$ of functions that are zero on the intersection of $\overline{\Omega}$ with the plane $\{x_3=0\}$, Γ is the latarel surface of Ω, k_1,k_2,σ are positive constants, but H is a nonlocal bounded linear operator from $L_p(\Gamma)$ to $L_p(\Gamma)$ such that for $p=1$ its norm is less than 1.

We show that there exists a two level iterative process that converges to the solution of (1). The first level consists of the Newton-type process

$$\begin{aligned} &\int_\Omega \big[k_1\langle\nabla v_{k+1},\nabla\eta\rangle + v_{k+1x_3}\eta\big]\,dx + \int_\Gamma \sigma\psi(v_k)v_{k+1}\,dS \\ &=\langle\langle F(v_k),\eta\rangle\rangle \quad \forall\eta\in V,\quad k=1,2,\dots\,, \end{aligned}$$

with approriate nonnegative function ψ and $F(v_k)\in\big(V\big)^*$. In its turn, the second level consists on iterations of the type

$$\begin{aligned} &\int_\Omega \big[k_1\langle\nabla(u_{k+1}+u_*),\nabla\eta\rangle + k_2(u_{k+1}+u_*)_{x_3}\eta\big]\,dx + \int_\Gamma \sigma\big[\mid u_{k+1}+u_*\mid^3 (u_{k+1}+u_*)\big]\eta\,dS \\ &=\int_\Gamma \sigma H\big[\mid u_k+u_*\mid^3 (u_k+u_*)\big]\eta\,dS + \langle\langle F_0,\eta\rangle\rangle \quad \forall\eta\in V,\quad k=1,2,\dots\,, \end{aligned}$$

with an appropriate $F_0\in\big(V\big)^*$.

REFERENCES

[1] K.Birgelis. Optimal control in models with conductive-radiative heat transfer. *Mathematical Modelling and Analysis*, **8** 1:1-12, 2003.

[2] P.E.Durot, O.Klein, J.Sprekels, F.Troeltzsch and I.Yousept. Optimal control of 3D-state-constrained induction heating problem with nonlocal radiaton effect. In: *Weierstraβ institute for Angevandte Analysis und Stochastic*, Preprint No. 1422 , 2009.

[3] M.Laitinen and T.Tiihonen. Heat transfer by conduction and radiation in grey materials. In: *University of Jyvaskyla*, Report No. B6/2000, 2000.

[1]This work was supported by the Latvian Council of Sciences under grant 09.1572 and by the ESF project No. 2009/0223/IDP/1.1.1.2.0/APIA/VIAA/008.