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SOME PROBLEMS OF SECOND-ORDER RATIONAL DIFFERENCE EQUATIONS[1]

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We consider the second-order rational difference equation

$$x_{n+1} = \frac{\alpha + \beta x_n + \gamma x_{n-1}}{A + B x_n + C x_{n-1}},\ n = 0, 1, 2, \dots , \tag{1}$$

where the parameters α, β, γ, A, B, C and the initial conditions x_{-1} and x_0 are nonnegative real numbers. These equations have been researched in the book [1]. The authors investigate the global stability character, the periodic nature, and the boundedness of solutions of a rational difference equation (1). The techniques and results that have been developed by the authors of the book [1] are developed to understand the dynamics of equation (1) are extremely useful in analyzing the equations in the mathematical models of various biological systems and other applications. It is an amazing fact that equation (1) contains, as special cases, a large number of equations whose dynamics have not been thoroughly explored yet.

In our talk we discuss about Open Problem 2.9.4 from the book [1]:

Open Problem. It is known that every positive solution of each of the following the equations

$$\begin{aligned} x_{n+1} &= 1 + \tfrac{x_{n-1}}{x_n}, \quad n = 0, 1, 2, \dots , \\ x_{n+1} &= \tfrac{1+x_{n-1}}{1+x_n}, \quad n = 0, 1, 2, \dots , \\ x_{n+1} &= \tfrac{x_n + 2x_{n-1}}{1+x_n}, \quad n = 0, 1, 2, \dots , \end{aligned} \tag{2}$$

converges to a solution with period-two: $\dots, \phi, \psi, \phi, \psi, \dots$. In each case, determine ϕ and ψ in terms of the initial conditions x_{-1} and x_0. Conversely, if $\dots, \phi, \psi, \phi, \psi, \dots$ is a period-two solution for one of equations (2), determine all initial conditions $(x_{-1}, x_0) \in]0, +\infty[\times]0, +\infty[$ for which the solution $(x_n)_{n=-1}^{\infty}$ converges to the period-two solution.

REFERENCES

[1] M.R.S. Kulenovič, G. Ladas. *Dynamics of Second Order Rational Difference Equations. With Open Problems and Conjectures*. Chapman&Hall/CRC, USA, 2002.

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