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AN ANALYSIS OF APPROXIMATION ON AN L-FUZZY SET BASED ON THE L-FUZZY VALUED INTEGRAL[1]

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In order to estimate the quality of approximation on an L-fuzzy set, we need an appropriate L-fuzzy analogue of a norm. In this talk we apply the L-fuzzy integral introduced in our previous papers to investigation of the error of approximation of a real valued function f on an L-fuzzy set E.

We assume that L is a completely distributive lattice, operations with L-fuzzy sets and L-fuzzy real numbers are based on the minimum t-norm, f is measurable with respect to a finite measure ν defined on a σ-algebra Φ of crisp sets, μ is the t-norm based extension of ν to an L-fuzzy valued measure on a tribe Σ of L-fuzzy sets and E is measurable with respect to μ, i.e. $E \in \Sigma$. The t-norm based construction of an L-fuzzy valued measure and L-fuzzy valued integral was considered in [1; 2].

Taking as a basis our previous works now we introduce an L-fuzzy valued norm defined by the L-fuzzy valued integral and describe the space $\mathscr{L}_1(E, \Sigma, \mu)$ of L-fuzzy integrable over $E \in \Sigma$ real valued functions. Notice that the norm $\|f\|_\mu$ in this case is characterized by an L-fuzzy real number, i.e. an order reversing left semi-continuous function taking values in L (for our purposes we use the fuzzy real line introduced by B.Hutton).

We show a possible application of the L-fuzzy valued norm described above in approximation theory. Being more precise, we use it to estimate on E the error of approximation $\mathscr{A}$ of a function $f \in \mathscr{L}_1(E, \Sigma, \mu)$:

$$e(f, \mathscr{A}, E) = \|f - \mathscr{A}f\|_\mu.$$

By a method of approximation we mean any operator

$$\mathscr{A} : \mathscr{L}_1(E, \Sigma, \mu) \to \mathscr{U},$$

where $\mathscr{U} \subset \mathscr{L}_1(suppE, \Phi, \nu)$ is a finite-dimensional space of functions used for approximation (it could be a space of polynomials or splines). Finally, we discuss the results of such analysis of approximation for some numerical examples.

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