

ON SMOOTHING PROBLEMS UNDER ADDITIONAL RESTRICTIONS¹

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Let X, Y be Hilbert spaces and assume that linear operator $T: X \rightarrow Y$, linear functionals $l_j: X \rightarrow \mathbb{R}$, $j = 1, \dots, m$, and $k_i: X \rightarrow \mathbb{R}$, $i = 1, \dots, n$, are continuous, functionals $l_1, \dots, l_{m-q}, k_1, \dots, k_n$ are linear independent and

$$l_{m-q+j} = \sum_{i=1}^n \phi_{ji} k_i, \quad j = 1, \dots, q.$$

For given vectors $u = (u_1, \dots, u_{m-q})$ and $v = (v_1, \dots, v_n)$, parameters $\varepsilon_i > 0$, $\omega_i > 0$, $i = 1, \dots, n$, and matrices $\Omega = \text{diag}(\omega_i)_{i=1, \dots, n}$, $\Phi = (\phi_{ji})_{j=1, \dots, q; i=1, \dots, n}$ we consider two following conditional minimization problems:

PROBLEM 1. (*the smoothing problem with obstacles*)

$$\|Tx\| \longrightarrow \min_{x \in X,} \\ A_1 x = (u, \Phi v), \\ |(A_2 x)_i - v_i| \leq \varepsilon_i, \quad i = 1, \dots, n,$$

PROBLEM 2. (*the smoothing problem with weights*)

$$\|Tx\|^2 + \|\Omega^{-1}(A_2 x - v)\|^2 \longrightarrow \min_{x \in X,} \\ A_1 x = (u, \Phi v).$$

Here the restrictions given by $A_1 = (l_1, \dots, l_m)$ describe the interpolating conditions and the restrictions given by $A_2 = (k_1, \dots, k_n)$ describe the smoothing conditions. This talk is devoted to the analysis of Problem 1 and Problem 2 in the case when some of functionals l_j , $j = 1, \dots, m$, depend on k_i , $i = 1, \dots, n$.

¹This work is partially supported by the projects 2009/0223/1DP/1.1.1.2.0/09/APIA/VIAA/008 and 2009/0138/1DP/1.1.2.1.2/09/IPIA/VIAA/004 of the European Social Fund and by the grant 09.1570 of the Latvian Council of Science.