

ON THE ERROR OF APPROXIMATION UNDER L -FUZZY INFORMATION¹

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Our main purpose is to develop methods of approximation under L -fuzzy information, i.e. information described by L -fuzzy sets, where $L = (L, \wedge, \vee)$ is a completely distributive lattice. It is closely related to our previous work on approximation theory in fuzzy context [1] and [2].

We consider the problem of approximation of an operator $B : X \rightarrow Y$ defined in a set X and taking values in a normed space Y under information given by an operator $A : X \rightarrow L^{\mathbb{R}^n}$, i.e. approximating Bx we suppose that an element $x \in X$ is given by an L -fuzzy set $Ax : \mathbb{R}^n \rightarrow L$.

By a method for solving this problem we mean any operator $\varphi : \mathbb{R}^n \rightarrow Y$ (in the classical case see e.g. [3]), which allows us to get an approximation of the exact value Bx for each $x \in X$. In this research we investigate the error of approximation by a method φ

$$e(\varphi, z) = \text{Sup}_{U_z} \|Bx - \varphi z\|_Y,$$

where $U_z \in L^X$ (namely $U_z x = (Ax)z$ for $x \in X$) and the supremum $\text{Sup}M : \mathbb{R} \rightarrow L$ of a bounded L -fuzzy set $M \in L^{\mathbb{R}}$ is defined by the formula

$$(\text{Sup}M)v = \bigwedge_{t < v} \bigvee_{u > t} Mu.$$

In particular we analyse the error of interpolating methods and spline methods.

REFERENCES

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