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ON MIXED INTERPOLATING – SMOOTHING SPLINES[1]

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The talk deals with mixed interpolating-smoothing (interpolating for a part of data and smoothing for the rest) splines in the abstract setting of a Hilbert space. Let X, Y be Hilbert spaces and assume that linear operators $T : X \to Y$, $A_1 : X \to I\!R^n$ and $A_2 : X \to I\!R^m$ are continuous. We consider the conditional minimization problem

$$(1) \qquad ||Tx|| \longrightarrow \min_{x \in H},$$

where restrictions given by A_1 (interpolating conditions) and A_2 (smoothing conditions) describe the set $H \subset X$.

We use the known results for the separate problems of pure interpolation and pure smoothing. For a given vector $\boldsymbol{u} \in I\!R^n$ in the case

$$(2) \qquad H = \{x \in X : \; A_1 x = \boldsymbol{u}\}$$

a solution of (1) is a spline from the space $S(T, A_1) = \{s \in X : \; < Ts, Tx >= 0 \text{ for all } x \in \operatorname{Ker} A_1\}$. Such spline is called the intepolating spline corresponding to the initial data $\boldsymbol{u}$. For a given vector $\boldsymbol{v} \in I\!R^m$ and parameters $\delta, \varepsilon_i > 0$, $i = 1, \ldots, m$, in the cases

$$(3)\ H = \{\, x \in X : \; |(A_2 x)_i - v_i| \le \varepsilon_i,\ i = 1, \ldots, m \,\}, \qquad (4)\ H = \{\, x \in X : \; \sum_{i=1}^{m} ((A_2 x)_i - v_i)^2 \le \delta \,\},$$

a solution of problem (1)-(3) (problem (1)-(4)) is a spline from the space $S(T, A_2)$. Such spline is called the smoothing spline corresponding to the initial data $\boldsymbol{v}$ and the smoothing parameter δ (parameters ε_i, $i = 1, \ldots, m$).

The purpose of this research is to investigate problem (1) with mixed interpolating and smoothing conditions: (2)-(3) or (2)-(4). We prove that solutions of such problems belong to the space of splines $S(T, A_1 \times A_2)$, where the operator $A_1 \times A_2 : X \to I\!R^{n+m}$ is given by the formula $(A_1 \times A_2)x = (A_1 x, A_2 x)$. Taking into account that in problems (1)-(2)-(3) and (1)-(2)-(4) the initial data $\boldsymbol{u}$ are interpolated and the initial data $\boldsymbol{v}$ are smoothed, we call solutions of these problems by mixed interpolating-smoothing splines.

In our research all central aspects of the variational theory of mixed interpolating-smoothing splines are discussed: the existence and uniqueness theorems are proved, the characterization via reproducing mappings is obtained.

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