MATHEMATICAL MODELLING OF PROPERTIES OF MAGNETORHEOLOGICAL SUSPENSIONS BY COMBINED DESCRIPTION OF PHYSICAL PROPERTIES AT MICRO AND MACRO SCALES

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Tasks and expected results of research subactivity

It was planned to further develop the methodology for modelling particle suspensions with magnetic properties using direct numerical simulation method.
For modelling of particle motion with carrier fluid corresponding algorithms should be elaborated and verification of results should be done focusing on two cases of spherical or rod-like particles. In magnetorheological fluid flow calculations at macro-scale should take into account that the external magnetic field causes anisotropy of the nature of the flow, which is related to the external field direction. Macroscopic flow simulation software should be further developed using the finite element method and the approximate modified Bingham law (with dependence on magnetic field) to comply with the anisotropy. Calculation of the magnetic interactions will be realized by improved approximate algorithms, as well as by improved finite element software to tabulate interaction parameters for magnetisable particles. An important result will be the numerical simulation methodology with capability to provide effective viscous stresses in a flow of the magnetorheological suspension accounting for the influence of magnetic force on the nature of flow.

1. Collaboration with mathematicians.

During the project the collaboration with mathematicians provided the following results:
1) clarifying the use of spherical harmonics for calculation of interaction between magnetized spheres;
2) accuracy verification for calculation of magnetic interaction by finite elements;
3) improving the numerical integration formulae;
4) developing a simplified interaction calculation approach in the case of magnetized rod-like particles;
5) the development of the theoretical models for hard particle description in fluid flow preventing the overlapping of particles.

2. Modelling of interactions between magnetized particles

Significant advances in the calculation of magnetic interactions are the following.

1. The usage of spherical harmonics to calculate interaction of several spheres is mastered in the case of constant magnetic permeability.
2. FEM (finite element method) software for the axially symmetric case is improved by introducing quadratic interpolation of elements. The use of quadratic interpolation enhances the accuracy of calculations at the same computation time (compared to linear interpolation).

3. FEM modelling software for the case of 3D magnetic interactions is also improved with the introduction of quadratic interpolation of elements with the possibility to pose periodic boundary conditions for the 3D case. These improvements allow to compute interactions inside particle chains in inclined field.

The Laplace's equation for scalar potential has to be solved.

$$\vec{\nabla}(\mu\,\vec{\nabla}\,\Psi) = 0$$

Nonlinear magnetization of magnetic material is described by Frohlich-Kenneley law:

$\mu = 1 + \frac{(\mu_{ini}-1)M_S}{(\mu_{ini}-1)H+M_S}$,

here μ_{ini} is so called initial permeability for small field values and M_S is the saturation magnetization. H is the magnetic field strength.

By calculating two sphere interactions at constant μ for various external field strengths, it was found that with reasonably selected mesh size relative error of force calculations is about 1.0-2.5%. The values of parameters were the following: $\mu = 100; 1000$, relative half gap between the spheres $\frac{d}{a} = 0.1; 1; 2$, the angle of field relative to the axis of symmetry $\phi = 0; \frac{\pi}{6}; \frac{\pi}{3}; \frac{\pi}{2}$. We can pose hypothesis that the relative error of magnetic interaction calculations in the case of non-linear magnetization will not exceed 2.5% if an adequate mesh size is used near the interacting particles.

The home-made software package MRmultiPart has been developed in the project. It is designed to calculate the magnetic interaction between several ferromagnetic particles of arbitrary shape. Nonlinear magnetization is described by Frohlich-Kenneley law and there is no restriction to incorporate another magnetization law, like one given by a spline function. The software package is programmed using C++ programming language. The executable file can be called up in Linux command line, so series of calculations are driven by Linux shell scripts. To prepare input FEM mesh, free software GMSH is also needed. GMSH could be downloaded from the internet (http://geuz.org/gmsh/). GMSH is also used for visualization of results (calculated physical fields).

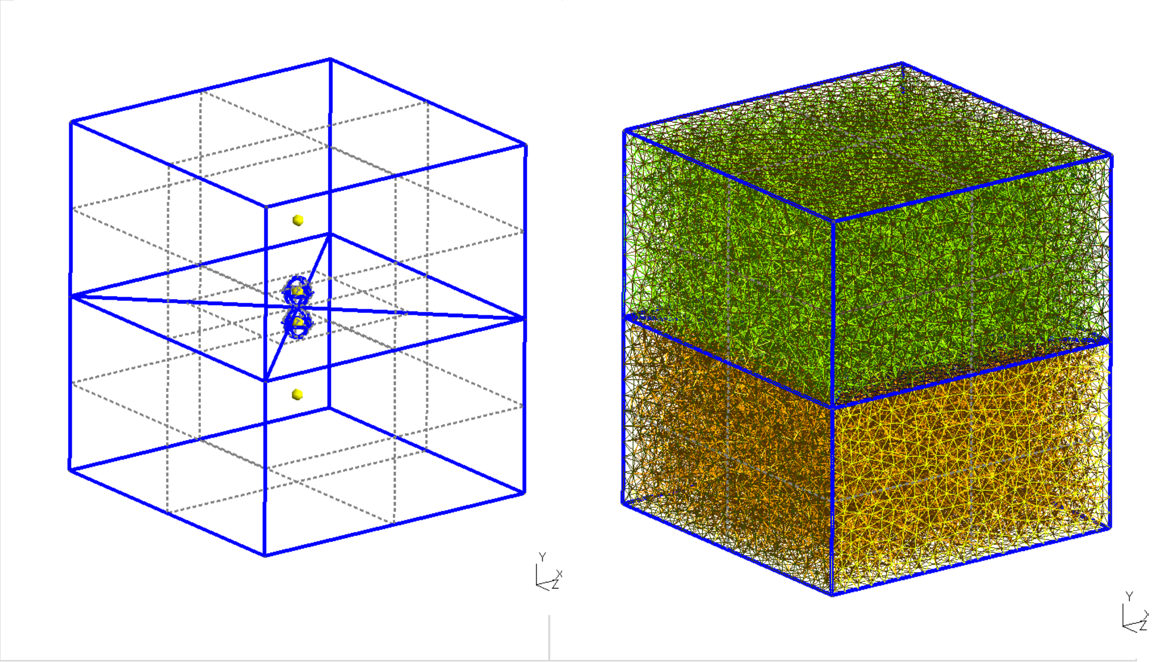


Fig.1. Geometry input for GMSH mesh generator	Fig.2. FEM 3D mesh, generated by GMSH using geometry input

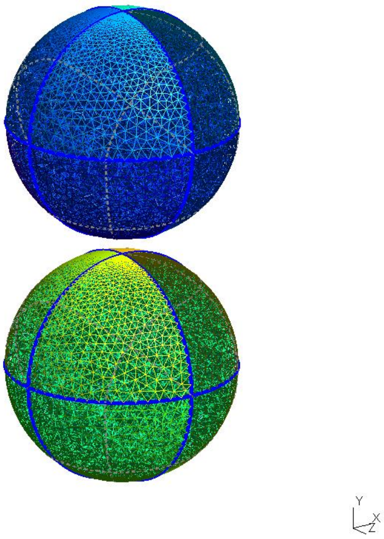
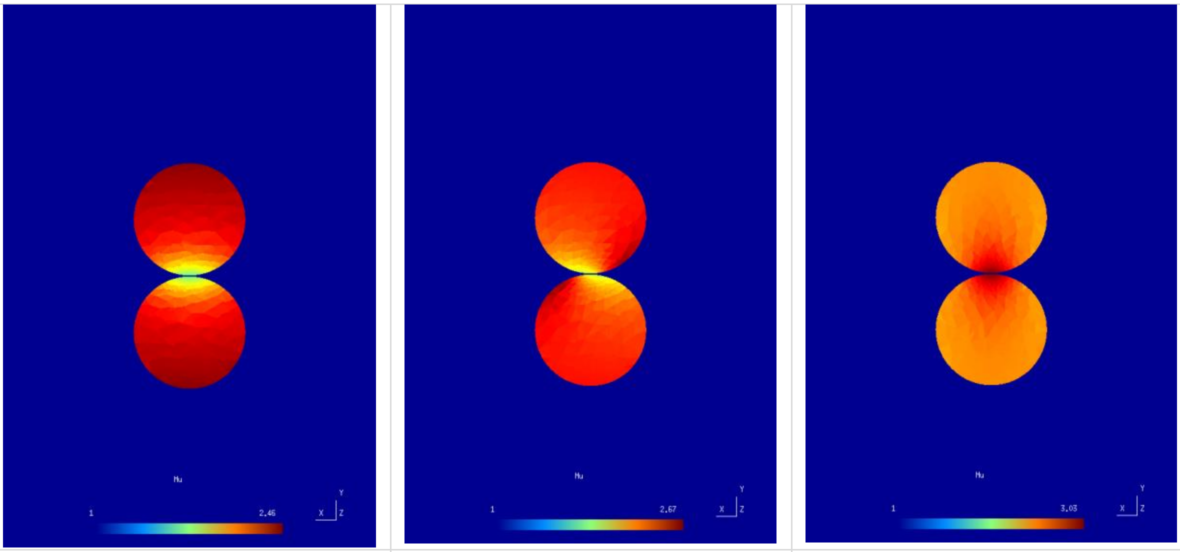


Fig.3. GMSH tetrahedral mesh, extended view of spheres



$\theta = 0$	$\theta = \frac{\pi}{4}$	$\theta = \frac{\pi}{2}$

Fig.4. Illustration of non-linear magnetization and the magnetic field saturation: relative permeability μ in the external magnetic field $\hat{H}_0 = 1.0$ at three different angles of magnetic field relative to the axis of symmetry $\theta = \left\{0, \frac{\pi}{4}, \frac{\pi}{2}\right\}$, the spheres are nearly touching (gap size $\hat{d} = 0.02$).

To compile the software package, UMFPACK (SuiteSparse) and BLAS libraries are needed (detailed description is given in the document "Programmu kompleksa MRmultiPart dokumentācija", in Latvian).

The software is able to calculate the forces for arbitrary shaped objects. For each case the separate mesh file must be created.

3. Modelling of magnetorheological suspension flow at micro level for spherical and rod-like particles.

When the project began, the home-made software for modelling of spherical suspension had already been developed. The magnetic interaction between the particles caused by external magnetization field was taken into account. A significant shortcoming of this model is that it is not able to predict magnetorheogical effect with sufficient accuracy.

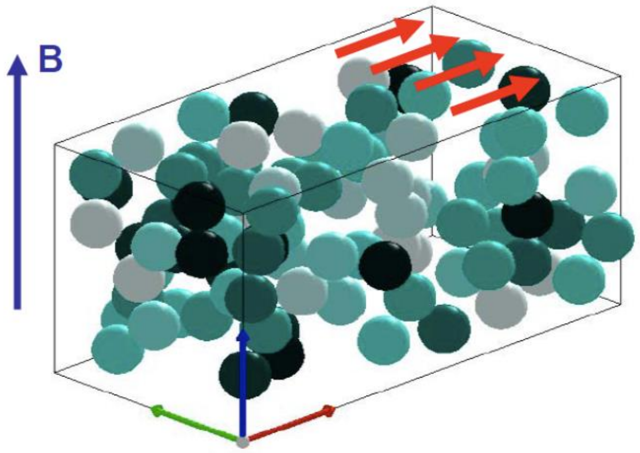


Fig.5. Particle suspension flow model

Simulations have proved that model of Fig.5 produces realistic picture of particle cluster formation.

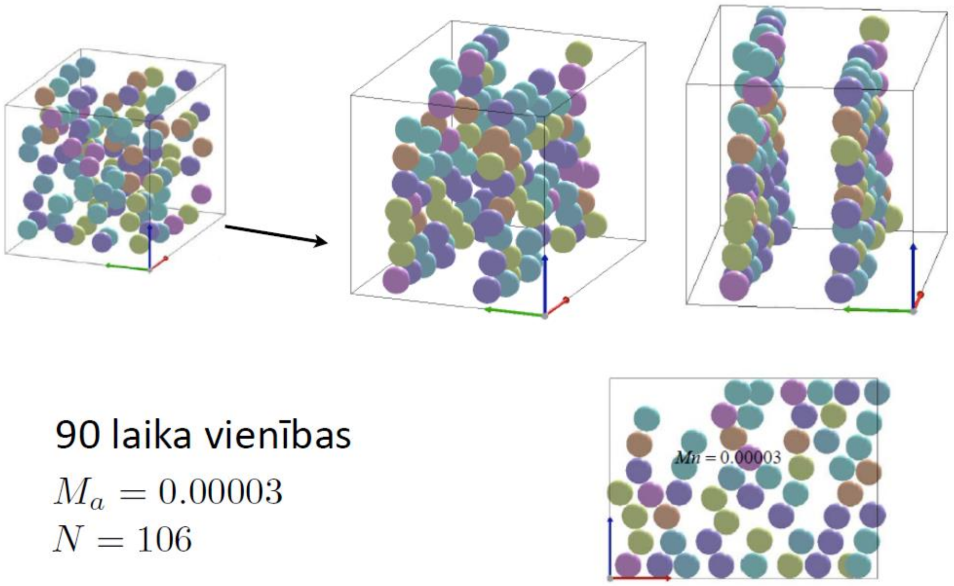


Fig.6. Formation of lamellar and short-time chain structures

Further development of suspension modelling was slowed down by absence of adequate methodology for calculation of effective flow stresses.

One of the aims of the project were direct modelling algorithms and software for the more general application of particle shape, focusing mostly on the needle (rod) magnetorheological particle suspension simulations.

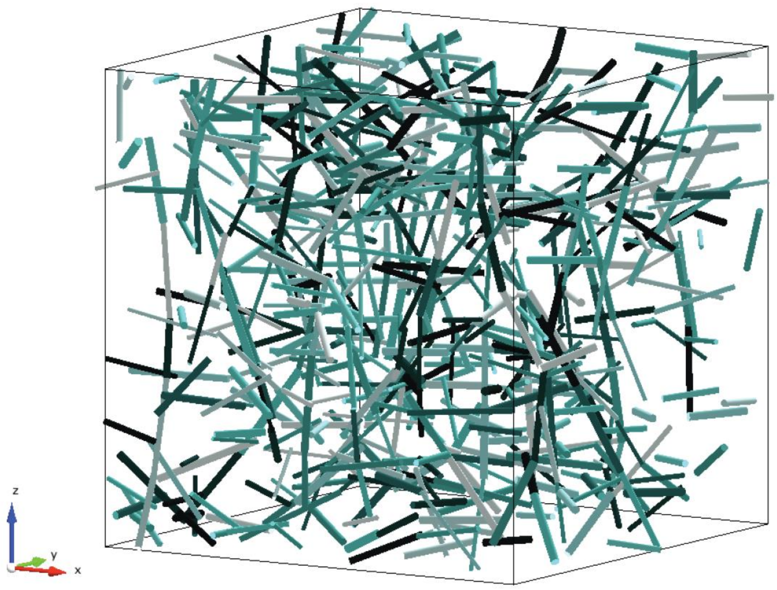


Fig.7. Flow of rod-like particle suspension, particle volume fraction ≈ 1,45%

Irregularly shaped particles have a greater opportunity to build agglomerates in relatively poor volume fractions of particles, but also the modelling is more tricky compared to spherical particles. The chosen model confirmed the correctness of the so-called Jeffrey's orbit calculated for each individual particle in shear flow, the calculated dimensionless value 61.8 corresponds very well to the theoretical value 61.9.

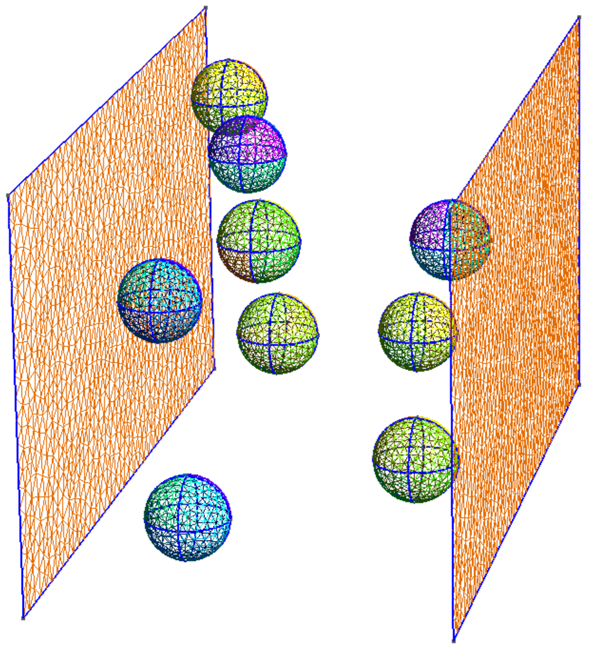


Fig.8. Example BEM mesh for calculation of effective stresses

During the project a hypothesis was proposed that for the calculation of the effective stresses the use of boundary element method (BEM) would be appropriate, possibly in combination with the fast multipole method. The initial inspection will be carried out in November 2012. BEM has the advantage that in the Stokes flow limit, we must determine unknown force source function distribution on particle and channel surfaces. Then the viscous stress contribution to the forces on walls is calculated, what then allows to calculate the effective forces. The methodology works equally well for particles of any form, if they have already a defined translation and rotation speeds.

4. Simulation of suspension flow at macro scale

Simulation of magnetoreological suspension as a viscoelastic fluid, which characteristics are determined by an external magnetic field, is realized at micro scale for axially symmetric geometry, which is typical in many industrial applications (e.g., car shock absorbers, etc.). The full 3D realization was performed as the test simulation because it is extremely computer resource consuming. Obtained results are not new and are not very interesting for pure research work but still form the base for research of applied industrial problems. During the project the master thesis (J.Cīmurs "*Magnetoreoloģisko suspensiju makroskopiskā skaitliskā modelēšana, izmantojot viskoelastīgu plūsmu modeļus*", 2010, University of Latvia, Faculty of Physics and Mathematics) were successfully defended on this subject.

For simulation at macro scale fluid flow is discussed in generalized Stokes flow approximation, assuming that the viscosity of the liquid depends on the velocity distribution, and thus it is the function of the coordinates. Flow is described by the stationary Stokes equations

$$0 = \vec{\nabla}\,\vec{\vec{\sigma}} + \vec{f}.$$

The velocity field-dependent viscosity is introduced for viscous stress tensor $\vec{\vec{\sigma}}$ using the Bingham law. Rate of flow is given by

$$\dot{\gamma} = \sqrt{\frac{1}{2}\dot{\varepsilon}_{ij}\dot{\varepsilon}_{ji}}\ ,$$

where $\dot{\varepsilon}_{ij}$ are components of rate of strain tensor. Bingham law gives relation between rate of shear and shear stress τ:

$$\dot{\gamma} = \begin{cases} 0 & ,\text{ja } \tau < \tau_0 \\ \dfrac{\tau - \tau_0}{2\,\mu_\infty} & ,\text{ja } \tau \geq \tau_0 \end{cases}$$

During simulations 3 methods as approximations of Bingham law were tested for the calculation of viscosity.

1st method (method without curve break)

$$\hat{\mu} = 1 + \frac{Bn}{\dot{\gamma}}\left[1 - e^{-\alpha\dot{\gamma}}\right]$$

2nd method (method with “break” at τ_0)

$$\hat{\mu} = \begin{cases} \hat{\eta} & ,ja\ \ \hat{\eta}\hat{\dot{\gamma}} < Bn \\ 1 + \dfrac{Bn}{\hat{\eta}\hat{\dot{\gamma}}}[\hat{\eta} - 1] & ,ja\ \ \hat{\eta}\hat{\dot{\gamma}} \geq Bn \end{cases}$$

3rd method (method with “break”)

$$\hat{\mu} = \begin{cases} \hat{\eta} & , ja \quad \hat{\dot{\gamma}} < \dfrac{Bn}{\hat{\eta} - 1} \\ 1 + \dfrac{Bn}{\hat{\dot{\gamma}}} & , ja \quad \hat{\dot{\gamma}} \geq \dfrac{Bn}{\hat{\eta} - 1} \end{cases}$$

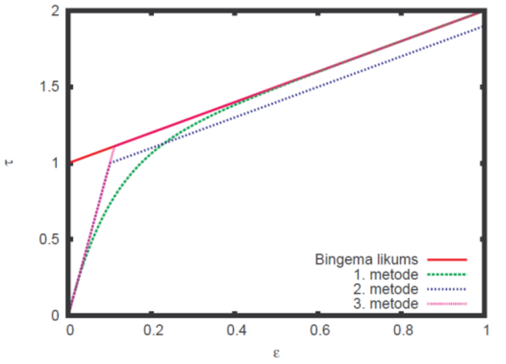


Fig.9. Viscous stresses as function of flow rate. Comparison of Bingham law with two approximations

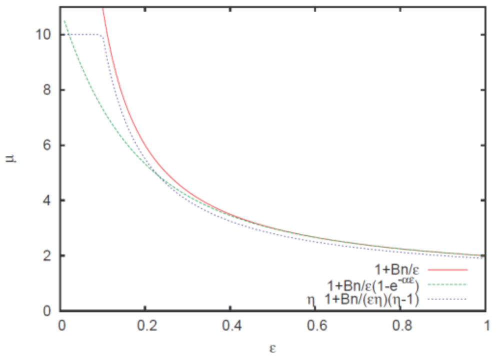


Fig.10. Viscosity coefficient as function of flow rate. Comparison of Bingham law with two approximations. ($Bn = 1, \hat{\eta} = 10, \alpha = 10$)

Methods were verified for cylindrical channel flow, testing 3 approximate Bingham law formulae. Comparison with analytical solution was carried out (Fig.11-14).

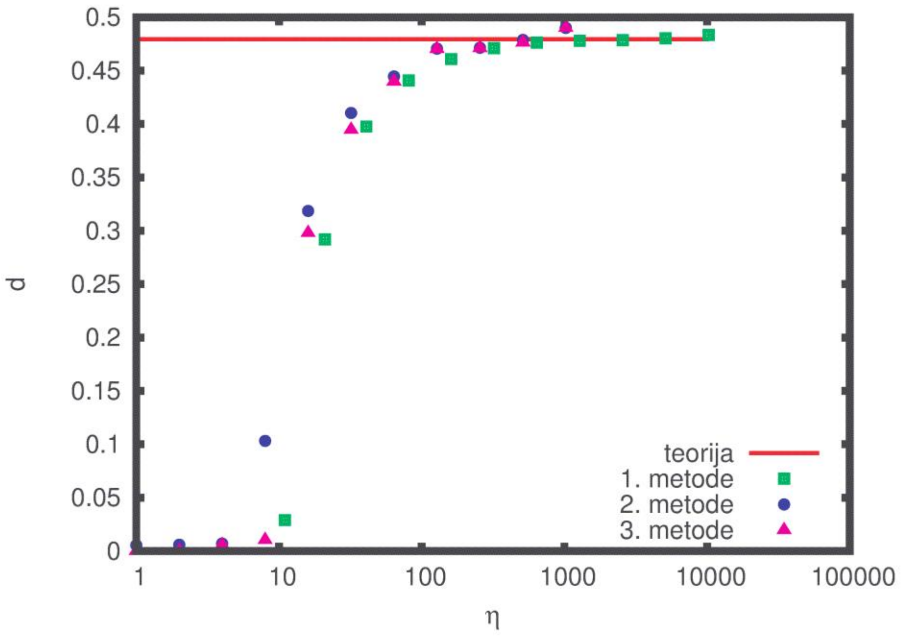


Fig.11. Size of flowing zone as function of parameter η , (msz = 0.02, Bn = 10)

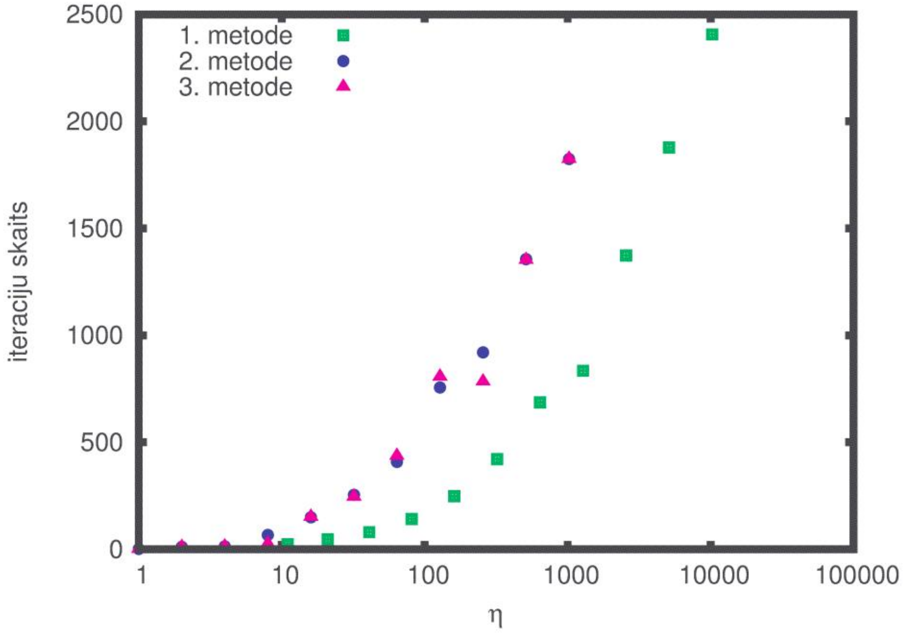


Fig.12. Number of iterations (rate of convergence) for 3 different methods

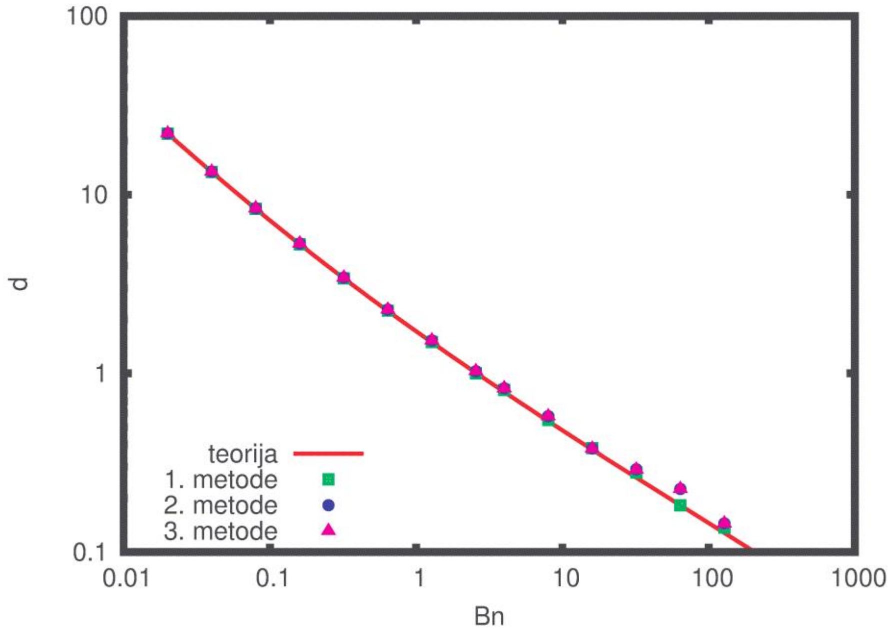
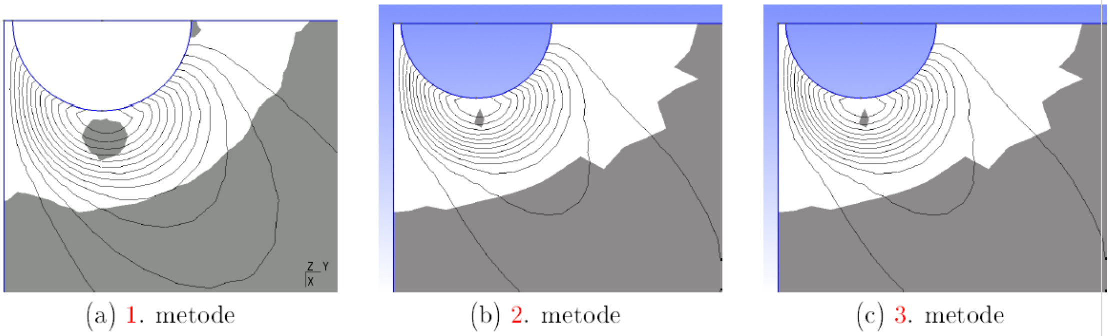


Fig.13. Comparison of approximations with theory at different Bingham numbersBn.



(a) 1. metode (b) 2. metode (c) 3. metode

Fig.14. Stagnation zone (grey) and flow lines for the sphere approaching the wall. Case of axisymmetric magnetic field, three approximations of Bingham law at $\hat{\eta} = 1000$

Flow around the sphere has shown (Fig.14) that 1st method produces slightly different results.
During project „intrinsic XFEM" approach was also tested but it produced no suitable results.

The main conclusions about the simulation methodology are:

1. Elastic and flowing regions are described by similar equations. Both behaviours should be connected at the boundary. Unfortunately, in order to make accurate estimates the jump of physical field should be introduced.
2. Bingham law can be approximated for modelling by the three methods described here, each with its own characteristics (advantages and disadvantages).
3. Three different approximate methods for Bingham law were developed and tested. It was obtained that the most commonly used 1st method (without curve break) is the fastest one, however, it not recommended for the calculations when magnetic field is applied to magnetorheological suspension.
4. The method without break was reformulated in such a way that different parameter was found. This parameter for wide variety of the Bingham number values provides the same accuracy.

Publications

[1] J. Cīmurs, J. Evertovskis, S. Lācis, *Dipole Aproximation Limits for Magnetic Interaction Forces Between Spheres*, Proceedings of the 6th International Scientific Colloquium, Rīga, 2010, pp 133-138;

[2] **D. Goško, S. Lācis**, *Magnetorheological Suspension Composed of Fiber Particles: Numerical Simulation of Anisotropic Behavior*, Proceedings of the 6th International Scientific Colloquium, Rīga, 2010, pp 335-340

Conference thesis

[3] **D. Goško, S. Lācis** *Numerical simulation of magnetorheological suspension composed of fiber particles*, 8th International pamir conference on Fundamental and applied MHD, Borgo, Corsica, France, September 5-9, 2011

University of Latvia Annual conference reports

[4] **J. Cīmurs, D. Goško, S. Lācis**. *Dipolu tuvinājuma pielietojamības robežas magnetizējamu daļiņu mijiedarbības aprēķinā*. University of Latvia 68th Annual conference, Physics section, February 4, 2010

[5] **D. Goško, S. Lācis**. *Magnetizējamu stieņveida daļiņu ansambļa uzvedība nesējšķidrumā. Modelēšanas iespējas.* University of Latvia 68th Annual conference, Physics section, February 4, 2010

[6] **D. Goško, S. Lācis**. *Magnetizējamu stienīšu mijiedarbības spēku modelēšana.* University of Latvia 69th Annual conference, Physics section, February 4, 2011

[7] **J. Evertovskis, S. Lācis**. *Magnetizējamu ložu ansambļa mijiedarbības spēku aprēķins, izmantojot multipolu izvirzījumu sfēriskās koordinātēs.* University of Latvia 69th Annual conference, Physics section, February 4, 2011

[8] **J.Evertovskis, S. Lācis**. *Multipolu rindas: pārbīdes formulas un izvirzījumu pielietošana magnētisma uzdevumos.* University of Latvia 68th Annual conference, Physics section, February 3, 2012